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ANALYSIS

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CONTENTS

THREE SHILLINGS AND SIXPENCE NET

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REPORT ON *ANALYSIS* 'PROBLEM' No. 12

"All swans are white or black." Does this refer to possible swans on canals on Mars?

By J. L. Austin

FIFTEEN entries were submitted, from four continents, including one from the U.S.S.R. It is agreeable to hear from such diverse places, and I print Mr. Mshvenieradze's contribution as of interest to Western readers.

The best entry was comparatively easy to select, and is printed below. Four or five others contained interesting points in places, but either these were not sufficiently central or else the answer did not cohere very well as a whole.

The point that I intended to raise—the limits of reference, in special connexion with the word "all"—was clearly seen by only about half of the entrants: it is admirably brought out by Mr. Matson. Some, taking for granted that the assertion *does* refer to "possible swans" (creatures that might be classified as swans) even on Mars, discussed the ways in which its truth could or could not be defended in the face of these disconcerting fauna: this is a different problem, and a more familiar one. Perhaps my expression "possible swans" was unfortunate—I meant by it "any swans there may eventually prove to be", not "any creatures that might be classified as swans".

Several entrants pointed out that there "cannot" be swans on Mars, because no creatures on Mars can be connected with creatures on Earth in the evolutionary process, which would however be necessary before they would be classified under the name of an Earth species. This had not occurred to me. I was only constructing a not-too-serious contemporary version of the text-book case, where the Ancient said "All swans are white" before Australia was discovered or even thought of: should we really assume without question that he was referring to (or, that his assertion referred to) "possible swans in undiscovered parts of the earth"?

My example touches only on the fringe of a large area of doubt: there seem, for instance, to be limits of reference with respect to time as well as to place—are we really referring to swans in the distant past or future when we say "All are" or "All do"?

Many entrants, including Mr. Matson, discussed *why* the assertion *could* not (be reasonably held to) refer to swans, if any, on Mars, the favourite answer being that it " is an inductive generalisation " and so could not sensibly refer to matters quite beyond the scope of the evidence available or investigated. I doubt whether this, even if it could be made more precise, could prove to be the whole story.

Some entrants discussed, though not very conclusively, the question: What is to be said about the truth or falsity of our assertion if swans which are not black or white do turn up in " unreferred-to " places? Perhaps the Ancient is unfairly treated if, on the discovery of black swans in Australia, his assertion is simply said to have been false: yet he must now withdraw or qualify it.

I. *By* WALLACE I. MATSON

I say, "All swans are either white or black ". Smith bets me a shilling that he can prove it is not so. Jones makes the same bet. Smith goes to Madagascar and returns with a red swan. Jones constructs a rocket ship, goes to Mars, and comes back with an ecru swan.

Clearly I owe Smith a shilling. But must I pay Jones?

I don't think so. Jones objects: " But here's a neither-black-nor-white swan! " I counter: " I didn't mean Martian swans, only terrestrial ones." " You didn't *say* that! " " No, nor did I even think of it. To quote Wittgenstein: ' Someone says to me: " Show the children a game." I teach them gaming with dice, and the other says " I didn't mean that sort of game." Must the exclusion of the game with dice have come before his mind when he gave me the order? ' " (*Phil. Inv.*, after 70.)

If this appeal to authority does not silence Jones, I may point out that when I made the bet I did so on the basis of a well-founded estimate of probability, which would *not* have been well-founded if it had been understood to include non-terrestrial fauna in its scope.

Jones: " Suppose our bet had been about exceptions to Ohm's Law—also an induction from terrestrial observations—and I had shown that it does not hold on Mars. Then by parity of reasoning you would have refused to pay up in that case also.

That would obviously be welshing. Therefore you are welshing on the swans."

But there is no parity of reasoning.

I draw, without replacement, a large number of balls from a Grecian urn, and find that all are either white or black. I conclude that probably the rest are all either white or black. Someone asks me to predict what are the colours of the possible balls in the butter-tub in the kitchen. My Grecian-urn induction provides *no* basis for an answer.

If, however, I have reason to believe that what is found to be true of balls in one urn is true of all balls—if, for instance, I know that there is only one ball factory, and the management never sorts the product—then, *and only then*, the experiment with the Grecian urn provides a basis for predicting the contents of the butter-tub; and my conclusion that all balls are either white or black would be understood not to be restricted to a specific urn.

The swan induction is an induction by simple enumeration, in which two characters are correlated without our being able to *explain* the correlation (I assume this, subject to correction by cycnologists). In the case of Ohm's Law, on the other hand, we understand *why* current varies directly with voltage, and consequently assert, without hesitation, that the generalization holds of Martian electrons.

This is not to say that the difference in the cases is absolute. (Does Mendel's Law hold of possible Martian sexual reproduction?) I do not want to deny (or affirm, either) that the most comprehensive theories rest ultimately on induction by simple enumeration. If the distinction made here is challenged on the ground that the swan " law ", Ohm's Law, and Maxwell's equations which explain it, are equally results of simple—terrestrial—enumeration, I reply that there is sufficient *direct* (astronomical) evidence that generalizations about fundamental properties of matter hold throughout the universe, *a fortiori* of Mars; but this is not so of generalizations which are about complex organisms and which are not embedded in theory.

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II. *By* V. V. Mshvenieradze[1]

The problem is ["All swans are white or black." Does this refer to possible swans on canals on Mars?][2]

LET us consider this problem purely from the formal point. Even if we could give an exact definition of the concepts: "white swan" and "black swan" (which is implied), then Θ might be formulated as follows: "Is (T) always true?" The answer would be undetermined in the case of our planet (as far as it is of an inductive character including the future) and undetermined in the case of Mars too.

If we had dichotomous division ("All swans are white or non-white"), it would be true for the earth and might be true for possible swans on canals on Mars.

The problem of truth or falsity cannot be decided within the pure logical sphere, by means of pure logical reasoning only. The best example of it, in my opinion, is Tarski's semantic conception of truth, that is admissible in logics, and yet it is not perfect: it cannot differentiate a true proposition from a false one. Tarski makes a very far reaching stipulation with his "*if* and *only if*".[3] Proceeding from the semantic conception of truth one cannot give a final quite definite answer, i.e. it is impossible to distinguish truth from falsity. Evidently, it is necessary to bring practice (as a criterion), the principle of verification into the very definition of truth. For it is impossible to discover a truth only by means of logical analysis.

When we say: (T), we tacitly imply then another proposition: "All swans *known to us* are white or black". Indeed, there was a time when black swans were unknown. Then the proposition T_1: ("All swans are white") appeared to be true. When black swans were discovered, then T_1 lost its apparent truth. (T) appeared to be true. Thus truth is a historic category. Truth is always concrete: it cannot be considered without reference to definite place and time, i.e. it should imply answers to the questions: "where?" and "when?"

(T) is an inductive conclusion. Bacon would call it—"inductio per enumerationem simplicem ubi non reperitur instantia contradictoria". Genuine inductive conclusion never

[1] Printed as received.—Editor.

[2] The expression taken into quotes we denote (T), the expression taken into square brackets—(Θ).

[3] The proposition "snow is white" is true, if and only if snow is white.

gives us quite reliable knowledge, because it extends and spreads our knowledge to things or classes of things that have not yet been investigated and we always can find out things which have other properties previously unknown to us.

Thus (T) seems to be an unwarranted conclusion; there is no sufficient ground to assert that *all* swans are white or black.

Even when we deal with a perfect induction [(T) only appears to be one] we never obtain new factual knowledge, though it is of scientific value.

(θ) is a problematic[1] statement, in which possibility is considered as probability, i.e. purely in a logical aspect.

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[1] Certainly, only the general thesis is problematic, but not the part of unconditional truth that it contains (that there exist and have existed white and black swans).

TRACTATUS 5.542

By Irving M. Copi

IN the *Tractatus* Wittgenstein held that " Propositions are truth-functions of elementary propositions " (5). He acknowledged that a difficulty is posed by " . . . certain propositional forms of psychology, like 'A thinks, that *p* is the case ', or 'A thinks *p* ', . . ." in which ". . . it appears superficially as if the proposition *p* stood to the object A in a kind of relation " (5.541). Wittgenstein's resolution of this difficulty was formulated in what J. O. Urmson describes in *Philosophical Analysis* (p. 133) as " a passage of almost impenetrable obscurity ".

> 5.542 But it is clear that "A believes that *p* ", "A thinks *p*", "A says *p* ", are of the form " '*p* ' says *p* ": and here we have no co-ordination of a fact and an object, but a co-ordination of facts by means of a co-ordination of their objects.

It will be convenient to follow Wittgenstein in using the letter " *p* " without quotation marks to refer to a fact (as in 5.43 " . . . a fact *p* . . .") and in using the letter " *p* " with quotation marks to refer to a proposition (as in 5.12 ". . . a proposition " *p* " . . .", and also in 5.123, 5.1241, 5.1311, 5.152, 5.44, 5.512, 5.513; but cf. 4.24, 5.141, and 5.541). For definiteness and simplicity attention will be confined here to atomic facts and elementary propositions. The fact *p* is a combination (2.01) or configuration (2.0272, 3.21) of entities or things (2.01, 2.0121, 2.0124, 2.013) or objects (2.01, 2.0141, 2.0272, 2.03, 2.031, 4.2211): $o_1, o_2, \ldots, o_k$. The proposition " *p* " is a combination (3.14) or configuration (3.21) or connexion or concatenation (4.22) of exactly the same number (4.0311, 4.04) of elements (3.2) or words (3.14, 4.026, 4.03) or signs (4.0312) or simple signs (3.201, 3.202, 3.21, 4.026) or symbols (3.31) or simple symbols (4.24) or primitive signs (3.26) or expressions (3.31, 3.318) or names (3.202, 3.26, 4.0311, 4.22, 5.55): $n_1, n_2, \ldots, n_k$. Each name n_1 signifies (3.323), represents (3.22, 3.221, 4.0312), stands for (4.0311), names or has for its meaning (3.203) the co-ordinated (2.1514, 5.526) or corresponding (2.13) object o_1. It is in virtue of this co-ordination of their constituent parts and the similarity of their (logical) form or structure (2.15, 2.171, 2.18, 3.21, 4.014, 4.12) that the proposition " *p* " asserts (4.03, 6.1264), communi-

cates (4.027, 4.03), describes (4.023), expresses (3.251), models (4.01), pictures (4.01, 4.011, 4.021, 4.032, 4.06), presents (4.021, 4.0311, 4.1), shows (4.461), signifies (4.061), or says (3.1432, 4.0621, 4.465, 5.542) the fact *p*.

Therefore, according to the *Tractatus* (especially 4.0311 and 4.04), whatever says a composite must be itself composite. But any fact *p* is composite. Hence any A that believes, thinks, judges, or says *p* must be composite. And since an object cannot be composite (2.021) but must be simple (2.02), A is not an object. An A that says *p* is a fact, the fact *that* A, and is therefore a configuration of objects $a_1, a_2, \ldots, a_q$. These considerations serve to explain Wittgenstein's assertion that in "A believes that *p*", "A thinks *p*", etc., we have no co-ordination of a fact and an object. A is not an object at all, hence not an object co-ordinated with the fact *p*.

There is more to be explained in the passage cited. Urmson suggests (p. 133) that Wittgenstein ". . . appears to assimilate belief to the uttering of a sentence, so that Jones's belief *is* the set of words he utters and ' Jones believes that *p* ' can therefore be said to be of the form ' " *p* " says *p* ' . . . ". But the word " belief " in ". . . Jones's belief . . ." is ambiguous, meaning either *what* he believes or his believing. (Different interpretations parallel different versions of the extensionality thesis set forth by Russell and Carnap. The first is like Carnap's suggestion on page 248 of *The Logical Syntax of Language* that " Charles says (writes, reads) A" can be translated into " Charles says 'A' ", and that " Charles thinks . . . A" can be translated into " Charles thinks 'A' ". The second is like Russell's suggestion in Appendix C, on page 661 of Volume I of the second edition of *Principia Mathematica*, that the series of words uttered is ". . . part of the series of events which constitutes the person ".) But it cannot be the former, because if Wittgenstein had meant that, he would have analysed "A thinks *p*" not into " ' *p* ' says *p* " but into "A says '*p*' ". The ambiguity of Urmson's formulation is thus easily resolved.

But I do not think that Wittgenstein would have agreed with Urmson's suggestion that ". . . Jones's belief *is* the set of words he utters . . .". For the *Tractatus* implies a quite clear distinction between a thought and a mere " set of words " (3.14, 3.141, 3.142), and even between a thought and a configuration of words: " In the proposition the thought is expressed perceptibly through the senses " (3.1) and " The sign through which we express the thought I call the propositional sign. And the proposition is

the propositional sign in its projective relation to the world' (3.12, but cf. 3.5 and 4).

One of the facts co-ordinated in "A thinks *p*" is the fact A (i.e., the fact *that* A). What is the other? In "A thinks '*p*'" or "A says '*p*'" the other fact would be the proposition or propositional sign '*p*'. But in "A thinks *p*" the other fact is *p* itself (i.e., the fact *that p*). Since the co-ordination between the two facts A and *p* is "by means of a co-ordination of their objects", it would seem that for A to think or believe *p*, among the objects $a_1, a_2, \ldots, a_q$ that constitute the fact A there must be *k* objects $a_{p1}, a_{p2}, \ldots, a_{pk}$ that are co-ordinated with the *k* objects $o_1, o_2, \ldots, o_k$ that constitute the fact *p*. Since it is this sort of co-ordination of the names $n_1, n_2, \ldots, n_k$ in "p" with the objects $o_1, o_2, \ldots, o_k$ in *p* that permits "p" to say p, Wittgenstein was justified in saying that "A believes that *p*", "A thinks *p*", "A says *p*", are of the form "'p' says p".

A possible objection to this interpretation of 5.542 is that people have false beliefs as well as true ones, and that only a true belief can be a belief in a fact. But the term 'fact' is ambiguous: there are both possible facts (Sachverhalten) and actual facts (Tatsachen). Thus "What is the case, the fact, is the existence of atomic facts. (Was der Fall ist, die Tatsache, ist das Bestehen von Sachverhalten.)" (2). A true belief is in an actual fact (Tatsache) whereas a false belief is in a possible but non-actual fact (Sachverhalt). The truth of the belief but not its actuality is involved in this distinction. For the same objects are combined in a possible atomic fact (Sachverhalt) as in the actual fact (Tatsache), and it is in terms of these objects that the belief statement must be analysed (" Every statement about complexes can be analysed into a statement about their constituent parts, and into those propositions which completely describe the complexes" (2.0201)). A true belief that *p* will be a configuration of (possibly non-verbal) names that corresponds to the actual configuration of the objects they designate; whereas a false belief will be a configuration of (possibly non-verbal) names that corresponds to a non-actual but merely possible configuration of the objects designated by them.

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IS EPIMENIDES STILL LYING?[1]

By William W. Rozeboom

A popular and not implausible gambit for resolving the Paradox of the Liar is to reason that the sentence whose truth or falsity is in question lacks those minimal semantic properties which are prerequisite for an entity to have truth-value at all. Thus faced with the expression

This sentence is false

we apparently escape the paradox by recognizing that it is simply not the case that all symbol-sequences are meaningful, and then arguing either that this expression is not a well-formed sentence at all (depending upon how we construe the syntax of English), or that even if it is *syntactically* a sentence, it is not a cognitively *meaningful* sentence. We may then propose that " This sentence is false " is neither true nor false, in the same way that we say that " dog " or "All blurps farble " are neither true nor false, without thereby abandoning any of the classical laws of logic. Similarly, Bar-Hillel[2] has recently contended that the symbol-sequence

The next sentence is true. The previous sentence is false

constitutes no genuine paradox because it is *statements*, not sentence-tokens, which are true or false, and that since a paradox would otherwise ensue, these symbol-sequences are not[3] statements.

While I am as appreciative as the next person of the need to convince ourselves that the languages we use, whether natural or formalized,[4] are free of logical anomalies, I fear that whatever is involved in the semantic paradoxes cuts too deeply to be effaced by this line of resolution. I shall present a form of the Liar's Paradox which does not appear to be touched by an appeal to " meaninglessness " or the like.

If there were no reason to heed the distinction between

[1] With apologies to Max Black.

[2] ANALYSIS, 18.1 (October, 1957), 1–6.

[3] Or " do not express ", or " are not tokens of ", or " do not make ", etc., depending upon how one construes the relation between sentence-tokens and statements.

[4] I am by no means convinced that contemporary formalisms have solved much with respect to the paradoxes. Syntactical Jim Crow legislation against sentence-forms which lead to paradox has been entirely too *ad hoc* for my liking.

statements (or alternatively, though not necessarily synonymously, *meaningful sentences* or *propositions*) on the one hand, and *sentence-tokens* on the other, I would present my version as follows. Consider the sentence P:

(P) There is a false sentence on this line.

We assume that a sentence may be true or false, though not both, but since " sentence " is a *syntactical* concept, that it need not be either. For simplicity, let us say that a sentence is " meaningless " when it is neither true nor false. Then, since P is the only sentence on its line, we observe that if P is true, it must be false; whereas if P is false, then it must be either true or meaningless. But if P is meaningless, then there is not a false sentence in the locus under consideration, and hence P is false. Thus given a scarcely questionable empirical premise, all three (logically exhaustive) alternatives, P is true, P is false, P is meaningless, lead to contradictions.

The force of this argument is attenuated, however, when we draw a distinction between statements and sentence-tokens. Since only sentence-tokens, not sentence-types, can have physical locus, it becomes apparent that the word ' sentence ' in P must be interpreted as " sentence-token ". But then two objections arise. (*a*) Not everyone would agree that sentence-tokens may be ascribed truth-values. This is in itself not too serious, since a suitable definition of truth and falsity as applied to sentence-tokens could be devised as soon as we agree on the relation between a sentence-token and whatever it is that has truth-value in the primary sense, and in any case, denial that sentence-tokens have truth-value impales one immediately on the third horn of the argument. However, (*b*) it is no longer clear how we pass from the statement, " There is not a false sentence(-token) in the locus under consideration ", which says nothing about which sentence-tokens its truth would falsify, to the conclusion that P is false. To reach this conclusion, we would need to spell out the details of the conditions under which a sentence-token has truth-value. Because of this weakness in the preliminary formulation, I will attempt to develop the present version of the Paradox in a slightly more sophisticated form.

Since I do not wish, at least for the present, to become embroiled in analysis of the relations holding among sentences, sentence-tokens, and statements, I shall simply assume that we know what these are, and that it is sometimes the case that a sentence-token *conveys* a statement. (Assertion that a sentence-

token " conveys " a statement is presumably elliptical for a rather complex statement about the effects of the token on a person with certain language habits.) I have chosen " conveys ", rather than " exemplifies ", " expresses ", or simply " is ", for expressing the relation between a sentence-token and a statement in order to remain as neutral as possible in regard to the nature of this relation. If the reader is suspicious of " statements " but has no qualms about " propositions ", he may substitute ' proposition ' for all appropriate occurrences of ' statement ' throughout this paper.

> In Figure 1, there is a sentence-token which conveys a false statement.

Figure 1. 'Figure 1' refers to the area enclosed in the box. The enclosed token of a symbol-sequence is designated in the text by 'S'.

Let us assume that the *expression*

(Σ) In Figure 1, there is a sentence-token which conveys a false statement

which we may designate by 'Σ', is a statement. By an " expression ", I mean whatever it is that is common to those occasions on which a given simple or compound symbol-type occurs with the same linguistic import, where this may, but need not, have cognitive significance. Thus occurrences of 'All greener house than ', though without cognitive content, involves something more, when produced or inspected by English-speaking persons, than when produced or inspected by persons to whom English is meaningless. The assumption that Σ is a statement will be re-examined later. For the moment, it may be supported by observing that if Figure 1 contained only the symbol-sequence, say, 'All snow is black ', Σ would be true, and would hence qualify as a statement. Presumably the only conditions under which Figure 1's *not* containing this symbol-sequence could change the statemental status of Σ would be that Σ contained a definite description, the satisfaction of which depended on the contents of Figure 1, and this, of course, is not the case. Presupposing, then, that expression Σ is also a statement, we are now to determine whether or not Σ is true.

Consider the sentence-token, which we may designate by

'S', contained in Figure 1. S must carefully be distinguished from Σ. The latter is (presumably) a *statement*, or is in any case an *expression*, and hence has no spatio-temporal locus. On the other hand, S is a sentence-*token* (as is also the symbol-sequence in the preceding paragraph by which Σ was *conveyed*) which may or may not convey Σ. In virtue of the topography of Figure 1 and the linguistic habits of the reader, we have three factual premises:

(1) S is a sentence-token, and moreover the only sentence-token, in Figure 1.

(2) If Σ is a statement, S conveys Σ.

(3) S conveys no statement other than Σ.

We have already assumed that

(4) Σ is a statement.

Since 4 allows us to *use* Σ cognitively, as well as to talk about it, semantical principles and the facts about the English language give two further premises:

(5) Σ is true only if it is the case that in Figure 1, there is a sentence-token which conveys a false statement.

(6) Σ is false only if it is not the case that in Figure 1, there is a sentence-token which conveys a false statement.

Suppose we entertain the hypothesis,

(H_1) Σ is true.

Then from H_1, 2, and 4, it follows that S conveys a true statement. But from H_1, 5, and 1, it also follows that S conveys a false statement and hence, from 3, that Σ is false. Considering

(7) No statement is both true and false

we see that H_1, 1–5, and 7 yield a contradiction; hence, by *reductio ad absurdum*, 1–5 and 7 entail that Σ is not true.

Alternatively, consider the hypothesis,

(H_2) Σ is false.

Then H_2, 2, and 4 imply that S conveys a false statement. From this, together with 1 and 6, it follows that Σ is not false. Thus by *absurdum*, 1, 2, 4, and 6 entail that Σ is not false. If we now adduce the Law of Excluded Middle,

(8) All statements are either true or false

we see, since 1–7 imply that Σ is a statement which is neither true nor false, that 1–8 are logically inconsistent.

But if we thus feel obliged to give up one or more of 1–8, which shall it be? Certainly not 1. The predicate, 'sentence-token' (interpreted in a suitably broad sense if necessary) is surely applicable to S, especially when 4 is accepted, and it is as indubitable as any empirical datum can be indubitable that there is no sentence-token other than S in Figure 1. As for 7 and 8, these are among the foundation principles of logical reasoning. So far as I am aware, no one has genuinely seen fit to question 7, especially since ambiguities in meaning may be written off to the statement-conveying properties of sentences rather than held against statements themselves. And while 8 has not been immune to criticism, its abandonment would necessitate too many profound changes in our habits of thought to make this a palatable solution except as a last resort.

What about 5 or 6? To be sure, it is not the case that expressions of the form

"p" is true (false) only if it is (is not) the case that p

where "p" is some English expression, are always true—for example

"All greener house than" is true only if it is the case that all greener house than

is not true, but simply meaningless. However, given the truth of 4, both 5 and 6 are statements and hence, by 8, are false—a highly counterintuitive possibility in view of the meaning of Σ —if they are not true.

It is likewise difficult to see how we could possibly deny 3. For S and the sentence-token by which the expression Σ was conveyed are tokens of the same sentence-type. While it is possible that S might fail to convey Σ at all, it would be highly arbitrary to claim that S does convey a statement, but that this differs from what is conveyed by the other occurrences of S's type in this paper. Such a move would be acceptable only if we could produce a sentence-token X, of a type *other* than that by which Σ was conveyed, and then find grounds on which to argue that S and X convey the same statement. Hence, it must surely be the case that S conveys no expression, and *a fortiori* no statement, other than Σ, whether Σ is itself a statement or not.

Thus we are left with 2 or 4 as our tentative source of error, and to be sure, neither of these carries quite as much intuitive

conviction as the others. But shall we deny 2 while retaining 4? If so, we are claiming that while " In Figure 1, there is a sentence-token which conveys a false statement " is indeed a statement, S does not convey it. But if the sentence-token enclosed in quotes in the previous sentence conveyed a statement to the reader—as indeed it must have if 4 is correct and the preceding sentence was read with understanding—then how can it be that S does not also convey this statement? It seems most implausible that of a set of similarly situated sentence-tokens, all of which exemplify the same sentence-type, the mere physical locus of one could emasculate it of its power to convey the statement conveyed by its brothers. If this be possible, the principles of language are more recondite than anyone has suspected.

But is the one remaining alternative, that Σ is not a statement, any more plausible than the ones already rejected? For if Σ is not a statement, then by 3, S does not convey a false statement, and hence from 1, *it is not the case that in Figure* 1, *there is a sentence-token which conveys a false statement.* The assumption that Σ is not a statement is apparently inconsistent in that it leads, by impeccable logic from what appear to be empirically true statements, to a conclusion which is the negation of Σ. Hence if we are to deny 4, we must be prepared to maintain either that the negation of an expression can be a statement even though the expression is not itself one, or that assertions about the existence of sentence-tokens which convey statements, at least as used in this paper, are generically meaningless, and that the fact we can apparently use such assertions in a meaningful way is only illusory.

As a matter of fact, the beginnings of an argument for the last alternative can be made by invoking a theory of *statement-types* (not to be confused with the " types " in the type-token distinction) or *language-hierarchies*, analogous to the ramified theory of predicate-types of *Principia* fame. I will not attempt to develop the theory here—it rapidly becomes complicated, and I would argue that it does not achieve the end for which it is invoked. For its crucial premise is that a statement about statements can only be about a proper subclass of statements (from which it would follow that expressions such as Σ and 3, which profess to quantify existentially over *all* statements, are syntactically ill-formed). This is a premise which I would brand as simply false, if not absurd. (Actually, in the form I have given it, it implies its own meaninglessness, and it is not at all certain that this can be circumvented.) For given, say, an expression E

which is syntactically ill-formed, it violates commonsense in the most outrageous way to claim that the statemental status of E can be described only by a transfinite set of assertions, each of which denies that E is a statement of a certain proper subclass of types. When I assert that the English expression, "All greener house than " is not a statement of *any* type, I feel just as confident that I have uttered a cognitively significant, and in fact true, statement as I do when I assert that not all snow is black. Similarly, once I convince myself that " Some swans are white " is a true statement (say, of type i) conveyed (to *me*, at time t) by the sentence-token

(T) Some swans are white

I am unable to see anything amiss about concluding that if s is a statement of *any* type conveyed (to *me*, at time t) by T, s is true.

It may indeed be true that expressions such as Σ (and hence also 3, since 1 and 3 entail the negation of Σ) are not genuine statements. However, the only reason which can at present be offered in support of such a counterintuitive belief is the dilemma posed by 1–8 when 4 is accepted. Until semiotical explorations more penetrating than any accomplished to date reveal some linguistic *principle* according to which Σ stands revealed as illegitimate, renunciation of 4 is no more a satisfactory solution to the incompatibility of 1–8 than is any of the other alternatives.

If the reader is expecting me to supply a happy ending to this unpleasant little tale, I am forced to disappoint him. While my feeling is that the difficulty lies somewhere between 4 and 8 (depending in part upon certain definitional stipulations), my present thoughts on the matter are too unclear to be of much help. But if I cannot supply a happy ending, at least I can draw a moral. The moral is that it is *not* the case that the semantic paradoxes are now only of historical or didactic interest. There are apparent inconsistencies in our use of language which are not going to be clarified until we achieve a much better understanding of what semantical concepts such as " statement ", " proposition ", " truth ", and the like are all about.

Note. Since this MS was submitted for publication, I have learned that what I thought to be a new vein of the Liar's lode was actually first explored by Ushenko[1] almost two decades ago, and has recently been mined fairly extensively by Toms.[2,3]

[1] *The Problems of Logic.* (Princeton, 1941), 78–80; and *Mind*, LXIV (1955), 543.
[2] *Philosophical Review*, LXV (1956), 542–547.
[3] *Philosophical Review*, LXVII (1958), 101–105.

However, as Encarnacion[1] has pointed out, Ushenko's formulation has technical errors, and Toms' efforts to tidy up the corners of the argument have been somewhat piecemeal and not wholly satisfactory:—(*a*) Toms' account of the Paradox is not fully formalized. In particular, all premises about the truth-conditions of sentences are suppressed (Premises 5 and 6 in the present version), and he has failed to indicate the rôle of Excluded Middle, which comes in essentially, either as a direct premise or used as a logical axiom along with sufficiently strong assumptions about truth-conditions. (In the present version, Excluded Middle is introduced as Premise 8, while the formal deductions do not depend on this somewhat controversial principle.) (*b*) Toms dismisses the argument from statement-types as irrelevant to his version of the Paradox. Unfortunately, his conclusion is not sound, though to show why, it would be necessary to recapitulate the details of his analysis. Actually, the theory of statement-types *would* provide a satisfactory resolution for the Paradox, were not the cure less tolerable than the disease. (*c*) In attempting to parry certain criticisms by Donnellan[2] along the type-token dimension, Toms[3] ascribes truth-values (and also Excluded Middle, if his argument is to work) to sentence-*types*. But taken literally, this will never do, for as Toms himself recognizes, the various tokens of a given sentence-type can be used in widely divergent senses.

It is also worthwhile, perhaps, to comment on Toms' attempted solution to the Liar's Paradox. He proposes[4] that the sentence in question ambiguously expresses two propositions. I am not wholly able to follow his argument; however, this line of resolution applied to the present version of the Paradox must suppose either (b_1) that S conveys at least one statement other than Σ, or (b_2) that Σ is itself ambiguous. The first alternative, denial of Premise 3, has already been discussed, so we may confine attention to the second. To begin with, it may be taken as a necessary condition for an expression to be a statement that the expression be unambiguous. (This assumption is not really essential, but it seems reasonable and simplifies the discussion.) Then b_2 entails that Σ is not a statement. Now, as seen earlier, Premises 1 and 3, together with denial that Σ is a statement, logically entail what appears to be the negation of Σ. Since the logical implicates of true statements are also statements (if this is not so, then we are *really* in trouble), and since pre-

[1] *Mind*, LXIV (1955), 99–100.
[2] *Philosophical Review*, LXVI (1957), 394–397.
[3] See p. 111, n. 3.
[4] See p. 111, n. 2.

sumably an expression must be a statement if its negation is to be a statement, b_2 leads to a dilemma: (b_{21}) either occurrences in the present context of the sentence-type, ' It is not the case that in Figure 1, there is a sentence-token which conveys a false statement ', do not convey the negation of Σ, or (b_{22}) Premises 1 and 3 not both true statements. Now if, as implied by b_{21}, the various tokens of a given sentence-type can differ in what they convey to an attentive, educated inspector of them within a narrowly circumscribed context of discourse, we might as well reject Premise 2 out of hand. But then we have also abandoned all hope for effective (i.e., semantically stable) discourse of any kind, unless we can adduce some *principle* according to which two tokens of the same symbol-type convey different expressions. As for b_{22}, Premise 1 seems unassailable, while 3 was defended earlier. I conclude that we are not going to find any cheap resolution of the Liar in terms of ambiguity. It should be noted, incidentally, that the classical laws of logic cannot be expected to hold for anything which is semantically ambiguous. Therefore, any semantic paradox which arises with respect to the truth-status of an entity to which the ascription of ambiguity is a live option merely shows that this entity is not the sort of thing to which the classical laws of logic apply.

The special virtue of the present formulation of the Liar's Paradox is that apart from the lack of symbolic abbreviation, the argument appears to be fully formalized. The trouble with a paradox which really *is* paradoxical is that we tend not to take it too seriously—since we know *something* is wrong, but don't offhand see what, it is easy to think we have merely unwittingly violated some rule already known to be necessary for correct argumentation. But in the present instance we have not a genuine paradox but simply eight premises which jointly entail a contradiction by the most respectable of logical principles; hence one or more of the premises must be written off as either false or cognitively meaningless. The difficulty is that it does not seem possible to discard any of these without jeopardizing belief either in one or more of what we have come to regard as linguistic principles, or in our ability to perceive reality. At least one premise has to go, but which one? And why?

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CAN THERE BE RANDOM INDIVIDUALS?

By NICHOLAS RESCHER

MATHEMATICIANS, logicians and others occasionally use the idea of " random individuals " in their expositions, and sometimes even have resort to these for serious theoretical purposes. The object of the present note is to demonstrate the logical impropriety of such procedure.

It will be convenient for my purpose to consider the idea of a " random " or " arbitrary " individual in the light of a passage from Professor I. M. Copi's fine textbook, *Introduction to Logic* (N. Y., Macmillan, 1953). The construction I shall place on this passage is that which, superficially, may seem the most natural to it. It is not, however, the one Copi intends, nor is it requisite, or even proper, for the system of natural deduction he is concerned to present. My purpose in using this passage is thus not to tax Copi with a logical blunder—he is quite innocent of that—but solely to afford myself the expository luxury of a convenient hook on which to hang the argument that I wish to present here.

> A geometer, seeking to prove that all triangles possess a certain property, may begin with the words: " Let ABC be any arbitrarily selected triangle." Then the geometer begins to reason about the triangle ABC, and establishes that it has the property in question. From this he concludes that *all* triangles have this property. . . . We wish now to introduce a notation analogous to the geometer's in talking about " any arbitrarily selected triangle ABC ". . . . We shall use the (hitherto unused) small letter " y " to denote any *arbitrarily selected* individual. We shall use it in a way similar to that in which the geometer used the letters "ABC ". Since the truth of any substitution instance of a propositional function follows from its universal quantification, we can infer the substitution instance which results from it by replacing " x " by " y ", where " y " denotes *any arbitrarily selected* individual . . . We may add . . . [the converse] principle to our list of elementary valid argument forms, stating it as: From the substitution instance of a propositional function with respect to the name of *any arbitrarily selected* individual one can validly infer the universal

quantification of that propositional function. This new elementary valid argument form may be written as:

$$\frac{\phi y}{\therefore (x)\phi x}$$

where " y " denotes *any arbitrarily selected individual.* [*Op. cit.*, pp. 293–295.]

Let us accept this discussion in a naive and literal way, and adopt the usage of special symbols that denote any " arbitrarily selected " individual.

At once trouble is upon us. For what can an " arbitrarily selected " individual be like? Consider a set $S = \{1, 2, 7, 8, 13\}$ Let " s " denote an " arbitrarily selected element " of S. Is s prime? Is $s < 9$? Is s even? Is s odd? Is $s = 7$? All of these questions, it is clear, must be answered negatively—otherwise s ceases to be " randomly " or " arbitrarily selected ". Thus down this road of " arbitrarily selected " individuals there lies only confusion and paradox. Somehow s must be an element of S. But it is a queer, shadowy sort of element indeed. For S has only 5 elements, yet s—though an element!—is not identical with any one of them. Here, surely, is a perfect paradox. s *is* an element of S. But it is not 1, not 2, not 7, etc. Yet 1, 2, 7, etc. are the *only* elements of S!

The reader will at once recognize this line of argument as familiar. It is, in fact, merely a crib of Berkeley's attack upon abstract ideas, upon " the general idea of a triangle—which is neither oblique nor rectangular, equilateral, equicrural nor scalene, but all and none of these at once " (Introduction to the *Principles*). Berkeley rejects an abstract idea of a particular thing, such as a triangle, as absurd because it demands an impossible coexistence of incompatible properties. And just this is the case also with " arbitrary " or " random " individuals.

The foregoing case against the idea of a " random " or " arbitrary " element of a set can be recast, with all due logical rigour, against " random " or " arbitrary " *individuals*. Let me return to the idea of an " arbitrarily selected individual " to be denoted by a special symbol, say " y ".

In any satisfactory system of quantificational logic we must inevitably have the inference of " universal instantiation ",

$$\text{(I)} \quad \frac{(x)\phi x}{\therefore \ \phi z},$$

i.e., if *everything* has a certain property, then any (particular) individual has this property. Let us now also assume the mode of argument characteristic of the "arbitrarily selected individual" y,

$$\text{(II)} \quad \frac{\phi y}{\therefore (x)\phi x},$$

i.e., if our "random" or "arbitrarily selected" individual has a certain property, then every individual must have this property.

From (II) we at once obtain, by *modus tollens*, the derived mode of argument,

$$\text{(III)} \quad \frac{(\exists x){\sim}\phi x}{\therefore \ {\sim}\phi y},$$

i.e., if some individual lacks a certain property, then the "arbitrarily chosen" individual y cannot have this property.

In any system of quantification logic adequate to universes of discourse that contain more than one thing, the following will be an accepted, or at least *acceptable* assertion:

$$(1) \quad (x)(\exists z){\sim}(x = z).$$

From (1) we at once obtain, by (I),

$$(2) \quad (\exists z){\sim}(y = z).$$

But now, by (III), (2) entails

$$(3) \quad {\sim}(y = y).$$

This paradoxical-seeming result leads to outright contradiction. For the logic of quantifiers cannot evade

$$(4) \quad (x)(x = x),$$

and this, by (I), yields

$$(5) \quad y = y.$$

But (5) contradicts (3). Thus the concept of a "randomly" or "arbitrarily selected" *individual* swiftly leads to outright contradiction (at any rate in universes with more than one thing).

The lesson of this line of argument is that any talk of "randomly" or "arbitrarily selected individuals" is thoroughly inept. When "s" is used to denote an "arbitrary element" of $S = \{s_1, s_2, s_3, \ldots\}$ what is intended is simply that a statement about s is to be true about every element of S: s is not an element of S — even though "$s \in S$" is true!—because a statement involving "s" is a shorthand synopsis of a multiplicity of statements, *viz.* the corresponding assertions regarding each particular s_i. Such an "s" is not a *thing* or "element" or

" individual " at all, it is a notational device: s is a surrogate for individuals, and not itself an individual. This is how it comes about that " $s = s_1$ " and " $s \neq s_1$ " are *both* false. There is not here some sort of infraction of the law of contradiction; for the former statement, without its misleading notation, amounts to " $(x)(x = s_1)$ " and the latter to " $(x)(x \neq s_1)$ ". And note that, once we cease to view y as an *individual*, rule (III) becomes wholly improper, for it licenses the inference from " $(\exists x){\sim}\phi x$ " to what is *now*, in effect, " $(x){\sim}\phi x$ ".

To regard a " random element " as an *element* or a " random individual " as an *individual* is to commit what Whitehead terms the " fallacy of misplaced concreteness " and involves what philosophers have come to call a *category mistake*. A statement like " ϕ y " does *not* say something about a peculiar " random individual " y : it says that the property ϕ characterizes every particular element of our universe of discourse. There are no " random " or " arbitrarily selected " individuals, just individuals. The "arbitrariness" or " randomness " resides not in individuals, but in the deliberate ambiguity of the notation by which reference to them is made. To talk of " random " or " arbitrarily selected individuals " is to reify a notational device. And this, in the present instance, is not merely unwarranted, it is demonstrably absurd.

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MORE ABOUT THE PARADIGM-CASE ARGUMENT

By H. G. Alexander

MR. HARRÉ (Analysis 18.4 pp. 94–6) is right in suggesting that the objections raised by Mr. Watkins (Analysis 18.2) against the Paradigm-Case Argument (PCA) are not, as they stand, conclusive. But the reasons Harré gives for criticizing Watkins are also unsatisfactory. Watkins' main argument is that if the connotation of a term p is determined by the chief items among its denotation, then many statements of the form ' This is p ' must be regarded as tautologies; if the denotation is determined by the connotation the PCA is useless. Harré

attempts to destroy the first horn of this dilemma. He writes, " If the term is defined by reference to a paradigm case this does not amount to the utterance of a tautology, for in a definition no statement is uttered and hence no tautologous statement. Ostensive definitions are verbal performances comparable to ' I name this child Willy '."

There are some situations for which Harré's account would be appropriate. Imagine a Lancashire firm of dyers faced with a demand for a shade of grey suitable for funerals. They devise a new dye of a particularly dismal hue which they proudly show to the public, proclaiming " This we call Manchester grey ", or " This is Manchester grey ". Or—to take a traditional example —one can imagine someone introducing the predicate *one-metre-in-length* by pointing to the bar deposited in the Pavillon de Breteuil at Sèvres and saying, " The portion of the bar between these two marks is one-metre-in-length ". It would be plausible to call such ' verbal performances ' stipulative ostensive definitions.

But it is very different with terms in current use, such as *red*. For here what is usually intended by the phrase *ostensive definition* is the procedure of teaching someone the meaning of the word *red* by pointing to pillar-boxes, buses, etc., saying " This is red ", and pointing to grass, gold, etc., saying " That is not red". In such cases the statements " This is red ", " That is not red " are clearly empirical assertions; they are neither tautologies nor are they analogous with baptismal formulae. Exactly the same applies to what are sometimes called *definitions-in-use*; the word *trick* is taught to someone by a player of a card game, who makes such statements as, " Now we've taken a trick ", " That's your trick ". To say these are not statements because their primary function is to enable someone to grasp the meaning of a word, would be like saying that the schoolmaster who teaches his pupils French by the direct method is not really asking them questions because he does not want the information conveyed in their answers. But this would be a ridiculous assertion.

It is clear that words like *red* are quite different from terms such as my imaginary *Manchester grey* or—if my simplified story were true—the term *one-metre-in-length*. These latter have their meaning tied to specific objects (not specific kinds of objects). When used of objects other than the standards, they are equivalent to ' like the standard object in colour ' or ' equal in length to the international prototype metre '. And therefore

it would be logically impossible for these terms to have a meaning if there were not, and never had been, any standard objects. But for words like *red* there are no individual standard objects. Smith may be taught its meaning by being shown pillar-boxes in Hampstead and Arsenal football jerseys, Jones by being shown buses in Manchester and Manchester United jerseys, and by the time they are adult they may both have completely forgotten which objects they once regarded as standard. 'This object is red' is not logically equivalent to 'this is the same colour as the pillar-box on the corner of Queen Street'; nor does it logically imply that there are or ever have been other red objects. (cf. Ayer, *Philosophical Essays*, p. 117). There are no standard red objects nor even standard kinds of objects. It is true that the dictionary refers to poppies and blood, but this is only because they have to choose some common type of object—poppies and blood have no epistemological primacy over pillar-boxes and buses.

Thus, as it stands, Watkins' main argument is invalid. One can deny that any particular statement 'This is red' is tautologous and yet use the PCA to establish the existence of red objects. For it is, I think, causally impossible that *red* should have a meaning if there are not and never have been any red objects. But although this example shows that Watkins overstates his case, his important contention remains unaffected. For what he has shown is that the PCA is applicable only if the predicate corresponds to what Locke would have called a simple idea, that is a predicate which can only be learned through ostensive definition. And the only cases in which it can be shown clearly that this condition holds are comparatively trivial ones such as *red*. Who ever wanted to deny the existence of red objects?

At the end of his paper Harré suggests that a "keen PCA-man" could have refuted Copernicus by pointing at the sun and saying "The sun goes round the earth". This is obviously wrong. For no one could claim that going-round-the-earth was a simple predicate. A more interesting example would be another medieval "keen PCA-man" who pointed to the surface of a large lake lying unruffled in the calm of a summer evening, while at the same time saying "This is flat". Many of us have learned to use the word *flat* with reference to such situations, and these associations are reinforced if we go on to be instructed in the use of spirit-levels. So could one perhaps go on to argue that the earth must be flat because the surface of a lake is a paradigm case of flatness? Obviously not. For

although many, and perhaps all, men are first introduced to the word *flat* when it is used in such contexts, they soon learn that there is another aspect of its meaning—the aspect which is implicit in the geometrical treatment of planes. And experience shows that it is extremely difficult, if not impossible, to regard the surface of the earth as planar. Although applied geometry may have arisen from observations interpreted in a language in which the surface of a lake was a paradigm case of flatness, it was realized that it was more convenient to make *flat* a theoretical term, even though this involved the apparent paradox of denying that the surfaces of lakes were exactly flat.

Now most of the cases in which the PCA has been applied are much more like this example than like the case of red—or the case of Manchester grey. For while it may be true that many of us are first introduced to phrases like ' of his own free will ' by hearing them applied to particular cases, it is also true that we soon go on to discover that such phrases figure in what may be called primitive theories. And in so far as a term figures in any type of deductive theory, it cannot be a simple predicate whose meaning can be *completely* conveyed by ostensive definition and *only* conveyed by ostensive definition.

In fact Watkins' dilemma can be replaced by another which avoids the objections raised against his. It is this. If the term ϕ is a simple predicate then the PCA can be used to show that there must exist, or have existed, objects which are ϕ; but because ϕ is a simple predicate it cannot figure in any deductions (except trivial ones such as the inference to ' there are objects which are ϕ or ψ ') and so no conclusions of philosophical importance can follow from the fact that there exist objects which are ϕ. If, on the other hand, the term is such that one can draw interesting conclusions from the proposition that there are ϕ objects, then ϕ cannot be a simple predicate whose meaning can only be conveyed ostensively. The PCA is therefore either trivial or inapplicable.

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